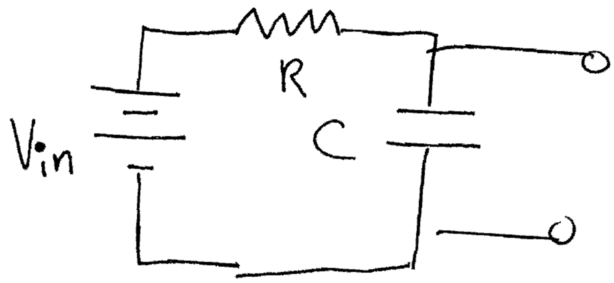
RC Filters:



$$V_{out} = \left(\frac{Z_c}{Z_c + R}\right) V_{in}$$

Z_c = impedance of the capacitor $= \frac{1}{i\omega C}$

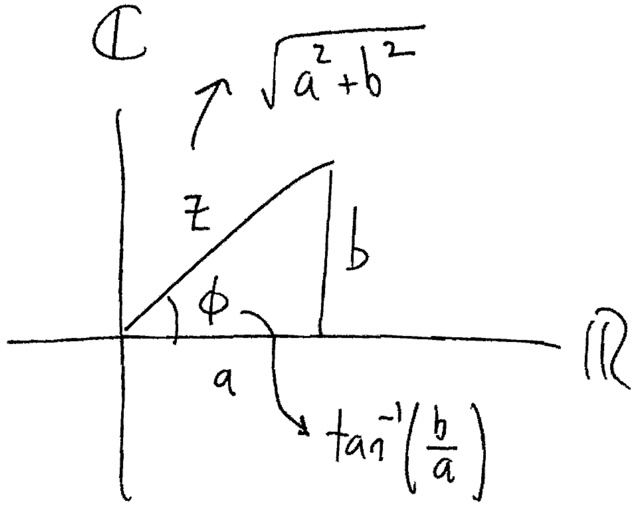
Recall that $\omega = 2\pi f$.

Also, the transmission function is defined by $A(\omega) = \frac{V_{out}}{V_{in}}$, which is just the divider ratio.

$$A(\omega) = \frac{Z_c}{Z_c + R} = \frac{1/i\omega C}{1/i\omega C + R}$$

$A(\omega)$ is complex and we want to find it's magnitude and phase.

Complex Plane :

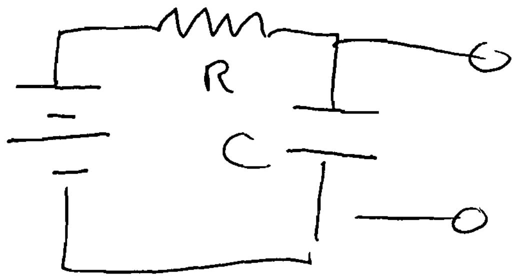


$$z = a + ib$$

$$|z|^2 = a^2 + b^2$$

$$\tan(\phi) = \frac{b}{a}, \quad \phi = \tan^{-1}\left(\frac{b}{a}\right)$$

I want to mention that both $z_1 = a + ib$ and $z_2 = a - ib$ have the same magnitude $\left(\sqrt{a^2+b^2}\right)$ because the magnitude of a complex number is defined by $|z|^2 = z^* z$, where z^* is the complex conjugate of z.



$$A(\omega) = \frac{1/i\omega C}{1/i\omega C + R}$$

The elegant way to solve all these transmission functions is to put them in $\frac{1}{a \pm ib}$ form, then use complex analysis on the denominator.

$$A(\omega) = \frac{1/i\omega C}{1/i\omega C + R} = \frac{1}{1 + i\omega RC}$$

Now the denominator looks like $a + ib$ where $a = 1$ and $b = \omega RC$.

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$$A(\omega) = \frac{1}{1+i\omega RC} = \frac{1}{\sqrt{1+(\omega RC)^2}\, e^{i\phi}}$$

$$\phi = ? \longrightarrow \phi = \tan^{-1}\left(\frac{\omega RC}{1}\right),$$ therefore

$$A(\omega) = \frac{1}{\sqrt{1+(\omega RC)^2}\, e^{i\tan^{-1}(\omega RC)}}$$

or

$$\boxed{A(\omega) = \frac{e^{-i\tan^{-1}(\omega RC)}}{\sqrt{1+(\omega RC)^2}}}$$